

TRANSPORT PLANNING AND MODELING

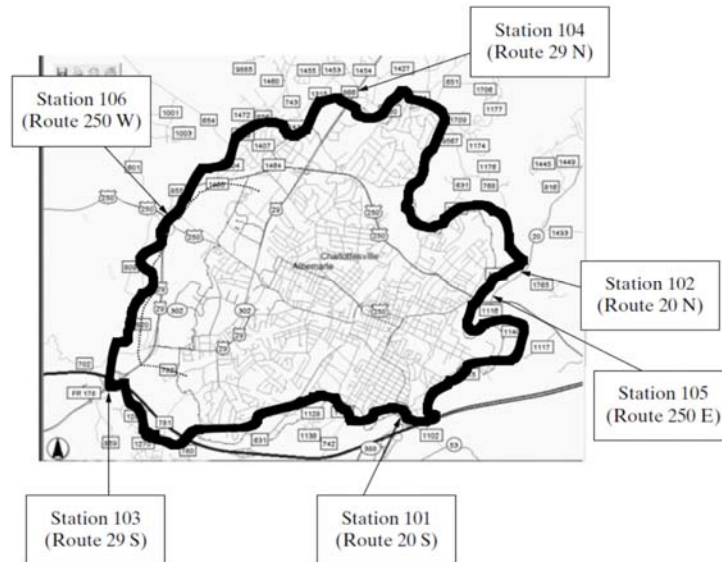
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INTRODUCTION

- Trip distribution is a process by which the trips generated in one zone are allocated to other zones in the study area.
- These trips may be within the study area (internal - internal) or between the study area and areas outside the study area (internal - external).
- For example, if the trip generation analysis results in an estimate of 200 HBW trips in zone 10, then the trip distribution analysis would determine how many of these trips would be made between zone 10 and all the other internal zones.
- In addition, the trip distribution process considers internal-external trips (or vice versa) where one end of the trip is within the study area and the other end is outside the study area.

- For example:

external stations for a study area boundary are depicted. If, for example, a trip begins somewhere south of the study area and ends in the center of the study area using Route 29, then an external – internal trip is defined that begins at external station 103 and ends in a zone located in the center of the study area.



Definitions and Notation

Origins	Destinations					$\sum_i T_{ij}$
	1	2	3	...j	...z	
1	T_{11}	T_{12}	T_{13}	... T_{1j}	... T_{1z}	O_1
2	T_{21}	T_{22}	T_{23}	... T_{2j}	... T_{2z}	O_2
3	T_{31}	T_{32}	T_{33}	... T_{3j}	... T_{3z}	O_3
...						
i	T_{i1}	T_{i2}	T_{i3}	... T_{ij}	... T_{iz}	O_i
...						
Z	T_{z1}	T_{z2}	T_{z3}	... T_{zj}	... T_{zz}	O_z
$\sum_i T_{ij}$	D_1	D_2	D_3	... D_j	... D_z	$\sum_{ij} T_{ij} = T$

- This is essentially a two-dimensional array of cells where rows and columns represent each of the z zones in the study area (including external zones)
- The cells of each row i contain the trips originating in that zone which have as destinations the zones in the corresponding columns.
- The main diagonal corresponds to intra-zonal trips.
- Therefore: T_{ij} is the number of trips between origin i and destination j ;
- The total array is T_{ij} or T ; O_i is the total number of trips originating in zone i , and D_j is the total number of trips attracted to zone j .

Origins	Destinations					$\sum_i T_{ij}$
	1	2	3	...j	...z	
1	T_{11}	T_{12}	T_{13}	... T_{1j}	... T_{1z}	O_1
2	T_{21}	T_{22}	T_{23}	... T_{2j}	... T_{2z}	O_2
3	T_{31}	T_{32}	T_{33}	... T_{3j}	... T_{3z}	O_3
...						
I	T_{i1}	T_{i2}	T_{i3}	... T_{ij}	... T_{iz}	O_i
...						
Z	T_{z1}	T_{z2}	T_{z3}	... T_{zj}	... T_{zz}	O_z
$\sum_i T_{ij}$	D_1	D_2	D_3	... D_j	... D_z	$\sum_{ij} T_{ij} = T$

- P_i is the number of trips produced or generated in a zone i and Q_j those attracted to zone j .
- We shall use lower case letters, t_{ij} , o_i and d_j to indicate observations from a sample or from an earlier study;
- Capital letters will represent our target, or the values we are trying to model for the corresponding modelling period.

- The matrices can be further disaggregated, for example, by person type (n) and/or by mode (k):
 - T_{ij}^{kn} are trips from i to j by mode k and person type n ;
 - O_i^{kn} is the total number of trips originating at zone i by mode k and person type n , and so on.

- p_{ij}^k is the proportion of trips from i to j by mode k ;
- c_{ij}^k is the cost of travelling between i and j by mode k .
- The cost element may be considered in terms of distance, time or money units.
- It is often convenient to use a measure combining all the main attributes related to the disutility of a journey and this is normally referred to as the generalised cost of travel.
- This is typically a linear function of the attributes of the journey weighted by coefficients which attempt to represent their relative importance as perceived by the traveller.

$$C_{ij} = a_1 t_{ij}^v + a_2 t_{ij}^w + a_3 t_{ij}^t + a_4 t_{ij}^n + a_5 F_{ij} + a_6 \phi_j + \delta$$

METHOD

- Growth factor (uniform, single constrained, double constrained)
- Synthetic or gravity model

Growth-Factor Methods

1. Uniform Growth Factor

If the only information available is about a general growth rate τ for the whole of the study area, then we can only assume that it will apply to each cell in the matrix:

$$T_{ij} = \tau \cdot t_{ij} \dots \text{for each pair } i \text{ and } j$$

Of course $\tau = T/t$, i.e. the ratio of expanded over previous total number of trips.

Example

Consider the simple four-by-four base-year trip matrix of Table 5.2. If the growth in traffic in the study area is expected to be of 20% in the next three years, it is a simple matter to multiply all cell values by 1.2 to obtain a new matrix as in Table 5.3.

Table 5.2 Base-year trip matrix

	1	2	3	4	\sum_j
1	5	50	100	200	355
2	50	5	100	300	455
3	50	100	5	100	255
4	100	200	250	20	570
\sum_i	205	355	455	620	1635

Table 5.3 Future estimated trip matrix with $\tau = 1.2$

	1	2	3	4	\sum_j
1	6	60	120	240	426
2	60	6	120	360	546
3	60	120	6	120	306
4	120	240	300	24	684
\sum_i	246	426	546	744	1962

2. Singly Constrained Growth-Factor Methods

- Consider the situation where information is available on the expected growth in trips originating in each zone, for example shopping trips.
- In this case it would be possible to apply this origin-specific growth factor (τ_i) to the corresponding rows in the trip matrix.
- The same approach can be followed if the information is available for trips attracted to each zone; in this case the destination-specific growth factors (τ_j) would be applied to the corresponding columns.
- This can be written as:

$$T_{ij} = \tau_i \cdot t_{ij} \quad \dots \quad \text{for origin-specific factors}$$

$$T_{ij} = \tau_j \cdot t_{ij} \quad \dots \quad \text{for destination-specific factors}$$

Table 5.4 Origin-constrained growth trip table

	1	2	3	4	\sum_j	Target O_i
1	5	50	100	200	355	400
2	50	5	100	300	455	460
3	50	100	5	100	255	400
4	100	200	250	20	570	702
\sum_i	205	355	455	620	1635	1962

This problem can be solved immediately by multiplying each row by the ratio of target O_i over the base year total (\sum_j), thus giving the results in Table 5.5.

Table 5.5 Expanded origin-constrained growth trip table

	1	2	3	4	\sum_j	Target O_i
1	5.6	56.3	112.7	225.4	400	400
2	50.5	5.1	101.1	303.3	460	460
3	78.4	156.9	7.8	156.9	400	400
4	123.2	246.3	307.9	24.6	702	702
\sum_i	257.7	464.6	529.5	701.2	1962	1962

3. Doubly Constrained Growth Factors

- An interesting problem is generated when information is available on the future number of trips originating and terminating in each zone.
- In transport these methods are known by their authors as Fratar in the US and Furness elsewhere.
- For example Furness (1965) introduced 'balancing factors' A_i and B_j as follows:

$$T_{ij} = t_{ij} \cdot \tau_i \cdot \Gamma_j \cdot A_i \cdot B_j$$

- or incorporating the growth rates into new variables a_i and b_j :

$$T_{ij} = t_{ij} \cdot a_i \cdot b_j$$

with $a_i = \tau_i A_i$, and $b_j = \Gamma_j B_j$

- This is achieved in an iterative process which in outline is as follows:
 1. Set all $b_j = 1.0$ and solve for a_i ; in this context, 'solve for a_i ' means find the correction factors a_i that satisfy the trip generation constraints;
 2. With the latest a_i solve for b_j , e.g. satisfy the trip attraction constraints;
 3. Keeping the b_j 's fixed, solve for a_i and repeat steps (2) and (3) until the changes are sufficiently small.
- This method can be said as bi-proportional algorithm' because of the nature of the corrections involved
- The most important condition required for the convergence of this method is that the growth rates produce target values T_i and T_j such that

$$\sum_i \tau_i \sum_j t_{ij} = \sum_i \Gamma_j \sum_i t_{ij} = T$$

Table 5.6 Doubly constrained matrix expansion problem

	1	2	3	4	\sum_j	Target O_i
1	5	50	100	200	355	400
2	50	5	100	300	455	460
3	50	100	5	100	255	400
4	100	200	250	20	570	702
\sum_i	205	355	455	620	1635	
Target D_j	260	400	500	802		1962

The solution to this problem, after three iterations on rows and columns (three sets of corrections for all rows and three for all columns), is shown in Table 5.7

Table 5.7 Solution to the doubly constrained matrix expansion problem

	1	2	3	4	\sum_j	Target O_i
1	5.25	44.12	98.24	254.25	401.85	400
2	45.30	3.81	84.78	329.11	462.99	460
3	77.04	129.50	7.21	186.58	400.34	400
4	132.41	222.57	309.77	32.07	696.82	702
\sum_i	260.00	400.00	500.00	802.00	1962	
Target D_j	260	400	500	802		1962

Note that this estimated matrix **is within 1%** of meeting the target trip ends, more than enough accuracy for this problem.

- A small study area has been divided into four zones and a limited survey has resulted in the following trip matrix:

	1	2	3	4
1	–	60	275	571
2	50	–	410	443
3	123	61	–	47
4	205	265	75	–

- Estimates for future total trip ends for each zone are as given below:

Zones	Estimated future origins	Estimated future destinations
1	1200	670
2	1050	730
3	380	950
4	770	995

- Use an appropriate growth-factor method to estimate future inter-zonal movements.
- *Hint:* check conditions for convergence of the chosen method first.

Advantages and Limitations of Growth-Factor Methods

- simple to understand and make direct use of observed trip matrices and forecasts of trip-end growth.
- This advantage is also their limitation as they are probably only reasonable for short-term planning horizons or when changes in transport costs are not to be expected
- Any error in the base-year may well be amplified by the application of successive correction factors. Moreover, if parts of the base-year matrix are unobserved, they will remain so in the forecasts. Therefore, these methods cannot be used to fill in unobserved cells of partially observed trip matrices.
- limitation is that the methods do not take into account changes in transport costs due to improvements (or new congestion) in the network. Therefore they are of limited use in the analysis of policy options involving new modes, new links, pricing policies and new zones.

Synthetic or Gravity Models

1. The Gravity Distribution Model

- They start from assumptions about group trip making behavior and the way this is influenced by external factors such as total trip ends and distance travelled.
- These models estimate trips for each cell in the matrix without directly using the observed trip pattern; therefore they are sometimes called synthetic as opposed to growth factor

$$T_{ij} = \frac{\alpha P_i P_j}{d_{ij}^2}$$

where P_i and P_j are the populations of the towns of origin and destination, d_{ij} is the distance between i and j , and α is a proportionality factor (with units trips·distance²/population²)

- Zone A connects to 4 other zones (B, C, D, E) with the number of origin in zone A is 25.000 trips
- Number of population in each zones and travel time between Zone A to Zone B, C, D, E is as follows:

Zone	Population (in thousand)	Travel time (in hour)
B	40	6
C	75	4
D	120	3
E	150	7

- Calculate the number of trip distribution from Zone A to Zone B, C, D, and E

- The model was further generalised by assuming that the effect of distance or ‘separation’ could be modelled better by a decreasing function, to be specified, of the distance or travel cost between the zones.

$$T_{ij} = \alpha O_i D_j f(c_{ij})$$

- where $f(c_{ij})$ is a generalised function of the travel costs with one or more parameters for calibration.
- This function often receives the name of ‘**deterrence function**’ because it represents the disincentive to travel as distance (time) or cost increases. Popular versions for this function are:

$$f(c_{ij}) = \exp(-\beta c_{ij}) \quad \text{exponential function}$$

$$f(c_{ij}) = c_{ij}^{-n} \quad \text{power function}$$

$$f(c_{ij}) = c_{ij}^n \exp(-\beta c_{ij}) \quad \text{combined function}$$

Table 5.8 A cost matrix and trip-end totals for a gravity model estimation

Cost matrix (minutes)					
	1	2	3	4	Target O_i
1	3	11	18	22	400
2	12	3	12	19	460
3	15.5	13	5	7	400
4	24	18	8	5	702
Target D_j	260	400	500	802	1962

given the information that the best value of β is 0.10. The first step would be to build a matrix of the values $\exp(-\beta c_{ij})$,

Table 5.9 The matrix $\exp(-\beta c_{ij})$ and sums to prepare for a gravity model run

$\exp(-\beta c_{ij})$					
	1	2	3	4	\sum_j
1	0.74	0.33	0.17	0.11	1.35
2	0.30	0.74	0.30	0.15	1.49
3	0.21	0.27	0.61	0.50	1.59
4	0.09	0.17	0.45	0.61	1.31
\sum_i	1.34	1.51	1.52	1.36	5.74

Base	1	2	3	4	\sum_j	Target	Ratio
1	253.12	113.73	56.48	37.86	461.19	400	0.87
2	102.91	253.12	102.91	51.10	510.04	460	0.90
3	72.52	93.12	207.23	169.67	542.54	400	0.74
4	31.00	56.48	153.52	207.23	448.23	702	1.57
\sum_i	459.54	516.45	520.15	465.87	1962.00		
Target	260	400	500	802			
Ratio	0.57	0.77	0.96	1.72			

Ranges (min)							
Cost	1.0–4.0	4.1–8.0	8.1–12.0	12.1–16.0	16.1–20.0	20.1–24	Sum
Trips	355.9	965.7	263.3	72.9	209.2	95.0	1962

Compared what if without value of β and using $T_{ij} = \frac{\alpha P_i P_j}{d_{ij}^2}$

- Consider a study area consisting of three zones.
- The data have been determined as follows: the number of productions and attractions has been computed for each zone by methods described in the section on trip generation, and the average travel times between each zone have been determined.

Table 12.9 Trip Productions and Attractions for a Three-Zone Study Area

Zone	1	2	3	Total
Trip productions	140	330	280	750
Trip attractions	300	270	180	750

Table 12.10 Travel Time between Zones (min)

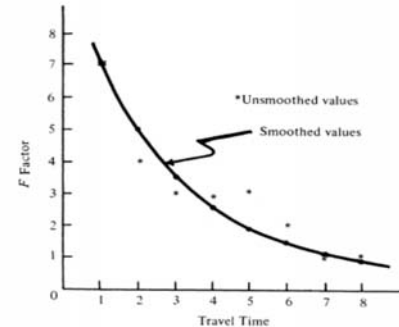
Zone	1	2	3
1	5	2	3
2	2	6	6
3	3	6	5

- Assume K_{ij} is the same unit value for all zones. Finally, the F values have been calibrated as previously described and are shown in Table 12.11 for each travel time increment.
- Note that the intra-zonal travel time for zone 1 is larger than those of most other inter-zone times because of the geographical characteristics of the zone and lack of access within the area.
- This zone could represent conditions in a congested downtown area.
- Determine the number of zone-to-zone trips through two iterations.

Table 12.11 Travel Time versus Friction Factor

Time (min)	F
1	82
2	52
3	50
4	41
5	39
6	26
7	20
8	13

Note: F values were obtained from the calibration process.



Calibration of F Factors

Table 12.10 and selecting the correct F value from Table 12.11. For example, travel time is 2 min between zones 1 and 2. The corresponding F value is 52.)

Use Eq. 12.3.

$$T_{ij} = P_i \left[\frac{A_j F_{ij} K_{ij}}{\sum_{j=1}^n A_j F_{ij} K_{ij}} \right] \quad K_{ij} = 1 \text{ for all zones}$$

$$T_{1-1} = 140 \times \frac{300 \times 39}{(300 \times 39) + (270 \times 52) + (180 \times 50)} = 47$$

$$T_{1-2} = 140 \times \frac{270 \times 52}{(300 \times 39) + (270 \times 52) + (180 \times 50)} = 57$$

$$T_{1-3} = 140 \times \frac{180 \times 50}{(300 \times 39) + (270 \times 52) + (180 \times 50)} = 36$$

$$P_1 = 140$$

Make similar calculations for zones 2 and 3.

$$T_{2-1} = 188 \quad T_{2-2} = 85 \quad T_{2-3} = 57 \quad P_2 = 330$$

$$T_{3-1} = 144 \quad T_{3-2} = 68 \quad T_{3-3} = 68 \quad P_3 = 280$$

The results summarized in Table 12.12 represent a *singly constrained* gravity model. This constraint is that the sum of the productions in each zone is equal to the number

Table 12.12 Zone-to-Zone Trips: First Iteration, Singly Constrained

<i>Zone</i>	<i>1</i>	<i>2</i>	<i>3</i>	<i>Computed P</i>	<i>Given P</i>
1	47	57	36	140	140
2	188	85	57	330	330
3	<u>144</u>	<u>68</u>	<u>68</u>	<u>280</u>	<u>280</u>
Computed A	379	210	161	750	750
Given A	300	270	180	750	

of productions given in the problem statement. However, the number of attractions estimated in the trip distribution phase differs from the number of attractions given. For zone 1, the correct number is 300, whereas the computed value is 379. Values for zone 2 are 270 versus 210, and for zone 3, they are 180 versus 161.

2. Singly and Doubly Constrained Models

- The need to ensure that the restrictions are met requires replacing the single proportionality factor α by two sets of balancing factors A_i and B_j as in the Furness model, yielding:

$$T_{ij} = A_i O_i B_j D_j f(c_{ij})$$

- In a similar vein one can again subsume O_i and D_j into these factors and rewrite the model as:

$$T_{ij} = a_i b_j f(c_{ij})$$

To create a doubly constrained gravity model where the computed attractions equal the given attractions, calculate the adjusted attraction factors according to the formula

$$A_{jk} = \frac{A_j}{C_{jk-1}} A_{jk-1} \quad (12.4)$$

where

- A_{jk} = adjusted attraction factor for attraction zone (column) j , iteration k
- $A_{jk} = A_j$ when $k = 1$
- C_{jk} = actual attraction (column) total for zone j , iteration k
- A_j = desired attraction total for attraction zone (column) j
- j = attraction zone number, $j = 1, 2, \dots, n$
- n = number of zones
- k = iteration number, $k = 1, 2, \dots, m$
- m = number of iterations

To produce a doubly constrained gravity model, repeat the trip distribution computations using modified attraction values so that the numbers attracted will be increased or reduced as required. For zone 1, for example, the estimated attractions were too great. Therefore, the new attraction factors are adjusted downward by multiplying the original attraction value by the ratio of the original to estimated attraction values.

Apply the gravity model (Eq. 12.3) for all iterations to calculate zonal trip interchanges using the adjusted attraction factors obtained from the preceding iteration. In practice, the gravity model becomes

$$T_{ij} = P_i \left[\frac{A_j F_{ij} K_{ij}}{\sum_j A_j F_{ij} K_{ij}} \right]$$

where T_{ij} is the trip interchange between i and j for iteration k , and $A_{jk} = A_j$ when $k = 1$. Subscript j goes through one complete cycle every time k changes, and i goes through one complete cycle every time j changes. This formula is enclosed in parentheses and subscripted to indicate that the complete process is performed for each trip purpose.

$$\text{Zone 1: } A_{12} = 300 \times \frac{300}{379} = 237$$

$$\text{Zone 2: } A_{22} = 270 \times \frac{270}{210} = 347$$

$$\text{Zone 3: } A_{32} = 180 \times \frac{180}{161} = 201$$

Perform a second iteration using the adjusted attraction values.

$$T_{1-1} = 140 \times \frac{237 \times 39}{(237 \times 39) + (347 \times 52) + (201 \times 50)} = 34$$

$$T_{1-2} = 140 \times \frac{347 \times 52}{(237 \times 39) + (347 \times 52) + (201 \times 50)} = 68$$

$$T_{1-3} = 140 \times \frac{201 \times 50}{(237 \times 39) + (347 \times 52) + (201 \times 50)} = 37$$

$$P_1 = 140$$

Make similar calculations for zones 2 and 3.

$$T_{2-1} = 153 \quad T_{2-2} = 112 \quad T_{2-3} = 65 \quad P_2 = 330$$

$$T_{3-1} = 116 \quad T_{3-2} = 88 \quad T_{3-3} = 76 \quad P_3 = 280$$

Table 12.13 Zone-to-Zone Trips: Second Iteration, Doubly Constrained

Zone	1	2	3	Computed P	Given P
1	34	68	38	140	140
2	153	112	65	330	330
3	116	88	76	280	280
Computed A	303	268	179	750	750
Given A	300	270	180	750	

- The results are summarized in Table 12.13. Note that, in each case, the sum of the attractions is now much closer to the given value.
- The process will be continued until there is a reasonable agreement (within 5%) between the A that is estimated using the gravity model and the values that are furnished in the trip generation phase.

When should a singly constrained gravity model or the doubly constrained gravity model be used?

- The singly constrained gravity model may be preferred if the friction factors are more reliable than the attraction values.
- The doubly constrained gravity model is appropriate if the attraction values are more reliable than friction factors.
- To illustrate either choice, consider the following example:

Table 12.15 Zone-to-Zone Trips: Singly Constrained Gravity Model

<i>Zone</i>	<i>I</i>	<i>2</i>	<i>3</i>	<i>Computed P</i>	<i>Given P</i>
1	235	118	47	400	400
2	235	118	47	400	400
3	<u>59</u>	<u>29</u>	<u>12</u>	100	100
Computed A	529	265	106	900	900
Given A	300	300	300	900	

Table 12.16 Zone-to-Zone Trips: Doubly Constrained Gravity Model

<i>Zone</i>	<i>I</i>	<i>2</i>	<i>3</i>	<i>Computed P</i>	<i>Given P</i>
1	133	133	133	400	400
2	133	133	133	400	400
3	<u>33</u>	<u>33</u>	<u>33</u>	100	100
Computed A	300	300	300	900	900
Given A	300	300	300	900	

- Table 12.15 is more likely to be accurate if engineering judgment suggests the occurrence of travel impedances and thus the friction factors are more accurate than trip attractions.
- Table 12.16 is more likely to be accurate if the attractions are more accurate than the friction factors.